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MATHEMATICAL MODELLING THROUGH DIFFERENCE EQUATIONS IN GENETICS

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ABSTRACT

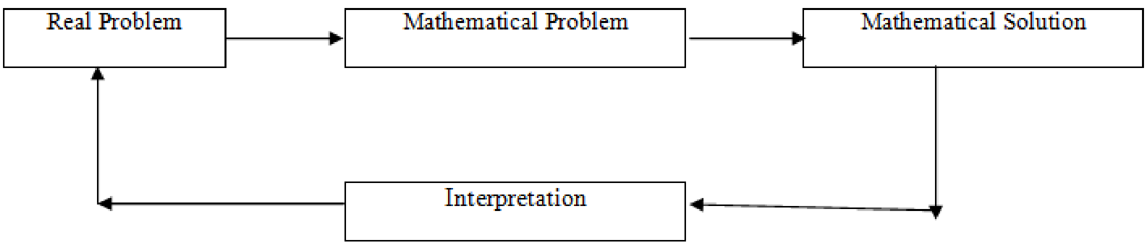
The real world is infinitely complicated. To penetrate that complexity using model building. We must learn to make reasonable, simplifying assumptions about complex process. Suppose the age of a father is four times the age of the son and after five years, the age of the father will be only three times the age of the son. We have to find their ages. So, let x be the age of the father and y be the age of son. Then the data of problem gives –

$$x = 4y, x + 5 = 3\,(y + 5) \qquad giving \qquad x = 40,\quad y = 10$$

These two equations give a mathematical model of the biological situation, so that the biological problem of ages is reduced to the mathematical problem of the solution of a system of two algebraic equations. The solution of equations is finally interpreted biologically to give age of the father and the son. In the same way to solve a given physical, biological or social problem, we first develop a mathematical model for it then solve the model and finally interpret the solution in terms of original problem.

I. INTRODUCTION

By definition any model is a simplification of the real world. Mathematical modeling essentially consists of translating real world problems into mathematical problems, solving the mathematical problems and interpreting these solutions in the language of the real world.



This is expressed figuratively by saying that we catch hold of the real world problem in our teeth dive into mathematical ocean, swim there for some time and we come out of the surface with the solution of the real world problem with us. This is actually one of the powerful benefits of a model – it forces you to think deeply about an idea.

II. MODIFICATION OF MODEL

It is quite often necessary to 'idealise' or 'simplify' the problem or approximate it by another problem which is quite close to the original problem and yet it can be translated and solved mathematically. In this idealisation, we try to retain all the essential features of the problem, giving up those features which are not very essential or relevant to the situation we are investigating. Considering the motion of planets, we may consider the planets and the sun as point masses and neglect their sizes and structures. Similarly for considering the motion of a fluid, we may treat it as a continuous medium and neglect its discrete nature in terms of its molecular structure. The justification for such assumptions is often to be found in terms of the closeness of the agreement between observations and predictions of mathematical models.

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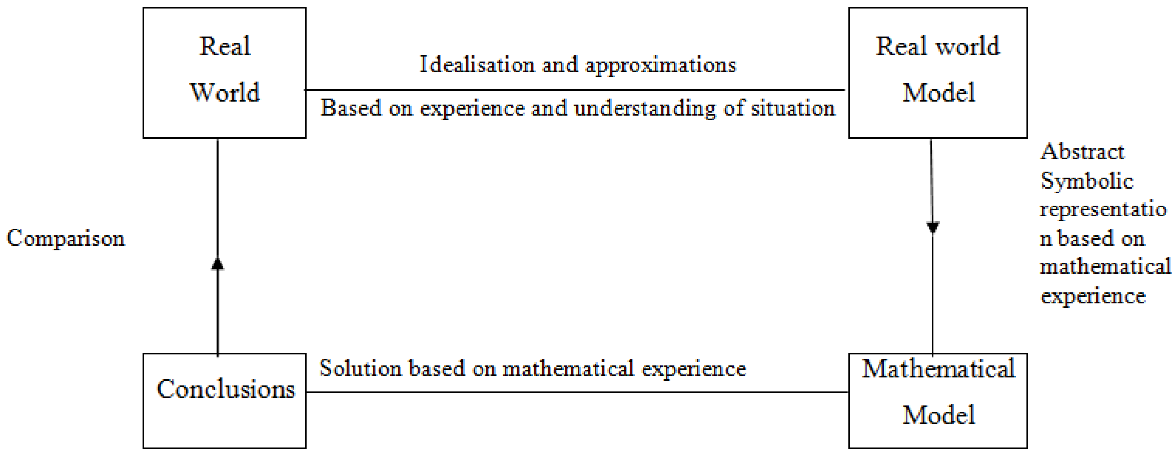

III. NEED OF DIFFERENCE EQUATION MODELS IN GENETICS

We need difference equation models when either the independent variable is discrete or it is mathematically convenient to treat it as a discrete variable.

Thus in Genetics, the genetic characteristic change from generation to generation and the variable representing a generation is a discrete variable.

IV. MATHEMATICAL MODELLING THROUGH DIFFERENCE EQUATION IN GENETICS

The Hardy Weinberg Law

The Hardy Weinberg Law (equation) is mathematical model in which allele frequencies in population remain constant from generation to generation.

Every characteristic of an individual, like height or colour of the hair, is determined by a pair of genes, one obtained from father and the other obtained from the mother. Every gene occurs in two forms, a dominant (say G) and a recessive (say g). Thus with respect to a characteristic, an individual may be dominant (GG), a hybrid (Gg or gG) or a recessive (gg).

In the n^{th} generation let the proportions of dominants, hybrids and recessives be p_n, q_n and r_n respectively so that

$$p_n + q_n + r_n = 1 \qquad ; p_n \geq 0, \quad q_n \geq 0, \; r_n \geq 0$$

We assume that the individuals in this generation mate at random. Now p_{n+1} = the probability that an individual in $(n+1)^{th}$ generation is a dominant (GG) = (Probability that this individual gets a G from the father) × (Probability that the individual gets a G from the mother)

$$= \left(p_n + \frac{1}{2}q_n\right)\left(p_n + \frac{1}{2}q_n\right)$$

$$p_{n+1} = \left(p_n + \frac{1}{2}q_n\right)^2 \qquad -(i)$$

$$Similarly, \quad q_{n+1} = 2\left(p_n + \frac{1}{2}q_n\right)\left(r_n + \frac{1}{2}q_n\right) \qquad -(ii)$$

$$r_{n+1} = \left(r_n + \frac{1}{2}q_n\right)^2 \qquad -(iii)$$

$$So\ that\ p_{n+1} + q_{n+1} + r_{n+1} = \left(p_n + \tfrac{1}{2}q_n\right)^2 + 2\left(p_n + \tfrac{1}{2}q_n\right)\left(r_n + \tfrac{1}{2}q_n\right) + \left(r_n + \tfrac{1}{2}q_n\right)^2$$

$$= \left(p_n + \frac{1}{2}q_n + r_n + \frac{1}{2}q_n\right)^2$$

$$= (p_n + q_n + r_n)^2$$

$$p_{n+1} + q_{n+1} + r_{n+1} = 1 \qquad \text{- (iv)}$$

The equations (i) – (iv) is a set of difference equations of the first order.

Similarly,

$$p_{n+2} = (p_{n+1} + \frac{1}{2}q_{n+1})^2$$

$$= \left((p_n + \frac{1}{2}q_n)^2 + \left(p_n + \frac{1}{2}q_n\right)\ (r_n + \frac{1}{2}q_n)\ \right)^2$$

$$= (p_{n+}\frac{1}{2}q_n)^2\ \left(p_n + \frac{1}{2}q_n + \frac{1}{2}q_n + r_n\right)^2$$

$$= \left(p_n + \frac{1}{2}q_n\right)^2$$

$$= p_{n+1}$$

$$\therefore p_{n+2} = p_{n+1}$$

and $q_{n+2} = q_{n+1}$, $r_{n+2} = r_{n+1}$ so that the proportions of dominants, hybrids, recessives in the $(n+2)^{th}$ generation are same as in the $(n+1)^{th}$ generation.

V. CONCLUSION

Thus in any population in which random mating takes place with respect to a characteristic the proportion of dominants, hybrids and recessives do not change alter the first generation. This is Hardy Weinberg Law after the mathematician Hardy and Geneticist Weinberg who jointly discovered it.

Similarly Mathematical modelling through difference equations can be used in other aspects also. For example in population dynamics, in economics, in geometry, in finance, in probability theory and so on.

It is much easier to solve the mathematical equations, provided we know how to formulate them and now to solve them. Moreover, quite often it is the only way to solve problems. Thus in measuring volume of blood in human body or mass of earth or temperature of the Sun or life span of a light bulb, the direct methods are impossible to use and mathematical modelling is the only alternative.

VI. WORKS CITED

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